Calculus Practice Problems

APEC Math Review 2021

# Part I: Practice problems

1. Find the limit of the function as using the definition of the limit of a function.
2. Prove the following statements using the definition of limit:
3. Show that is a continuous function for using the definition of limit.
4. Consider the function . Show that is a convex function.
5. Derive the formula for differentiation of an inverse function, namely .
6. Consider the function defined by . Show that is a concave function.
7. Derive the Hessian for the function and check that Young’s Theorem holds.
8. Derive the Jacobian of the function
9. Derive the Jacobian of the vector field (multi-valued function)
10. Totally differentiate the following function with respect to :

where and are functions of defined by and .

# Part II: Proofs

1. Prove the calculus criteria for a concave function of 1 variable, namely:

Let be a continuous function on an open interval (a,b). Then is concave on (a,b) if and only if for all , .

(Hints: you need to prove both directions of the biconditional statement. For one, try to rearrange the inequality to get a statement of the form and take the limit to get a derivative. For the other, substitute in a new defined by into one of the expressions in the inequality.)

1. Prove the sum law of limits: if and both have limits at , then:
2. Prove that . (hint: recall the precise definition of a differential)
3. Consider the CES production function defined as . Show that for positive , this function converges to the Cobb-Douglas production function as . (hint: take the log of the function and use L’Hospital’s Rule).